A.7 OC680 Assignment 7, for extra credit, due Friday Mar. 23

1. **Sheared convection**

   Here you will use your existing stratified shear flow function to investigate the effect of background shear on convective instability. Sheared convection is an important aspect the dynamics of thunderstorms and of upper ocean response to intense surface forcing (e.g. hurricanes). You'll begin by testing the code by reproducing the results we obtained analytically in the first week of the course. You'll then repeat the analysis with a background shear flow added, and find something interesting.

   (a) **Test your software.**

   Using the Matlab script given below along with your subroutine for stratified shear flow, *plot growth rate versus wavenumber and Rayleigh number for a flow with

$$0 < z* < 1\,; \quad \Delta* = 0.05\,; \quad U* = 0\,; \quad \nu* = Pr = 7; \quad \kappa* = 1;$$

$$B^* = -RaPr\beta; \quad \beta = z*; \quad B^*_{z*} = -RaPr \cdot 1$$

   Compute with both frictionless and rigid boundaries. In both cases, buoyancy can obey fixed-buoyancy conditions. *Check to make sure that the critical values of Ra and k delivered by your code match the theoretical values.

   The following script will guide you through most of this, but you must insert the call to your function for stability analysis of stratified shear flow with the appropriate inputs and outputs. If you prefer to write your own script that's fine.

```
% Hmwk 7, project 1A
clear
close all
fs=18;
lw=1.6;

% set parameters
Pr=7;        % Prandtl number
irigid=0;    % boundary conditions (0=frictionless, 1=rigid)

% define z values
del=.05;
z_st=[del:del:1-del]';

% define profiles
beta=z_st;
beta_zst=ones(size(z_st));
U_st=zeros(size(z_st));

% analytical results for critical Rayleigh number and wavenumber
if irigid==0
    Ra_c=(27/4)*pi^4; % frictionless boundaries (as derived in class)
    k_c=pi/sqrt(2);
    bcw='ff'
elseif irigid==1;
    Ra_c=1708;  % rigid boundaries (Kundu 11.3)
    k_c=3.12;
```

```
        bcw='rr';
    end
    bcb='cc';

    % ranges for loops over k and Ra
    ks=[0:.2:12];nks=length(ks)
    Ras=10.^[2:.2:5];nRa=length(Ras);
    l_st=0;     % 2D modes only

    % loop over k and Ra
    for i=1:nks
        k_st=ks(i);
        for j=1:nRa
            Ra=Ras(j);
            Bz_st=-Ra*Pr*beta_zst;

            %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
            % put call to your stratified shear flow routine here
            [s_st] = ...
            %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

            sig(i,j)=real(s_st);
        end
        disp([num2str(i/nks) '  done'])     % indicate progress
    end

    % plot growth rates along with analytical values for the critical Ra and k
    figure
    contourf(ks,Ras,sig',max(sig(:))*[0:.05:.95]);shading flat
    hold on
    plot([k_c-1 k_c+1],Ra_c*[1 1],'k','linewidth',lw)
    plot(k_c*[1 1],Ra_c*[1/1.5 1.5],'k','linewidth',lw)
    set(gca,'yscale','log')
    colorbar
    ylabel('Ra','fontsize',fs,'fontangle','italic','fontweight','bold')
    xlabel('k*','fontsize',fs,'fontangle','italic','fontweight','bold')
    title('Benard convection scaled growth rate', 'fontsize',fs,'fontweight','normal')
    set(gca,'fontsize',fs-2)

    print('-djpeg','hmwk7_1a')
```

(b) The sheared case.

First, investigate the possibility that oblique modes are the most unstable by doing the following. *Modify the script you used for part A so that the Rayleigh number is fixed at 1000, and loop instead over both k^* and ℓ^*. The range $0 \leq k^*, \ell^* \leq 4$ is sufficient. How do growth rates vary as the angle of obliquity is increased?

Now apply a uniformly sheared background flow $U^* = RePr\, z^*$, setting $RePr = 20$. Run the script again. *Now how do growth rates of oblique modes compare with those of 2D modes?

*On the basis of the result above, what effect does the shear have on the growth rates of *two-dimensional* ($\ell^* = 0$) modes? Does it increase the growth rates, decrease them or leave them unchanged? *What is the angle of obliquity for the fastest-growing mode? *Given the information above, could you have *predicted* the angle of obliquity for the fastest-growing mode?

2. **Instabilities of the Eady model**

   Based on section 7.6, implement a solution procedure

$$[\sigma, \hat{q}, \hat{w}, \hat{b}] = \mathscr{F}(z, U_z, B_z, f, k, l)$$

   (a) Baroclinic modes

   Test your code by computing growth rates as a function of k with $\ell = 0$ and $Ri = 100$. Compare the growth rate and wavenumber of the fastest-growing mode with (7.8.21) and (7.8.22) or (7.8.23).

   (b) Symmetric modes

   Computing growth rates as a function of ℓ with $k = 0$ and $Ri = 0.75$. Does the growth rate increase monotonically with ℓ as in the analytical solution for symmetric instability? If not, why not?