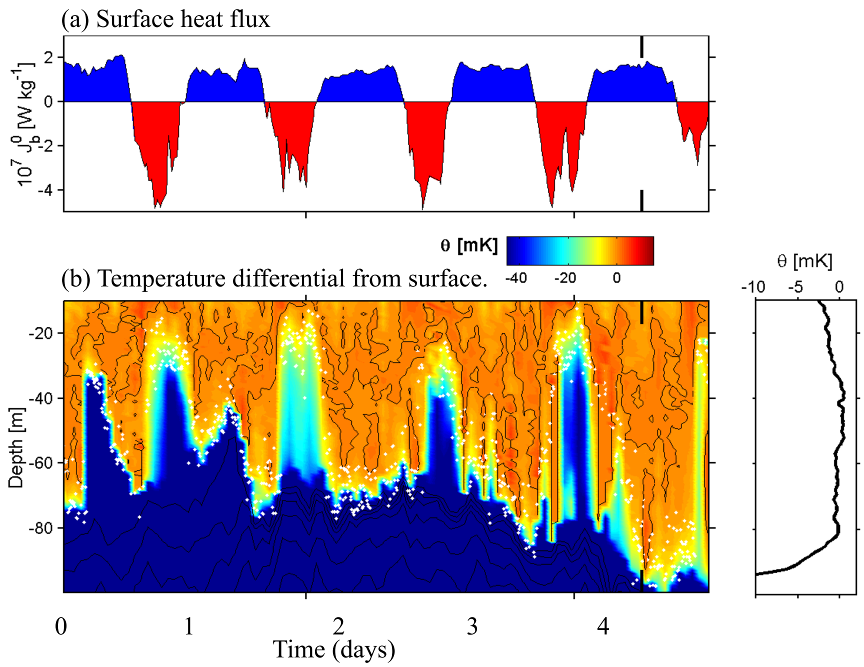

Figure A.1: The diurnal cycle of upper ocean convection (courtesy J. Moum). For reference, a temperature change $\Delta T = 2mK$ is equivalent to a relative density change $\Delta\rho/\rho = 10^{-6}$.

OC680 Homework #2 Due Tuesday Jan. 23

1. **Benard convection**

   (a) Given

   $$\sigma^2 + (\nu+\kappa)K^2\sigma + \nu\kappa K^4 + B_z\cos^2\theta = 0 \tag{A.0.2}$$

   as derived in class, show that

   $$\frac{\partial\sigma}{\partial\cos^2\theta} > 0$$

   [Hint: You don't have to solve the quadratic equation to do this. Just differentiate each term.] What assumptions do you have to make about σ and B_z for this to be true? Write a brief (1-sentence) justification for each assumption. At what value of $\cos^2\theta$ will σ be greatest (if all other parameters are fixed)?

   (b) Minimize the function $(\tilde{k}^{*2}+n^2\pi^2)^3/\tilde{k}^{*2}$ with respect to $\tilde{k}^{*2}$. Give both the minimum value, and the value of $\tilde{k}^{*2}$ at which the minimum occurs, as functions of n . Show that the critical Rayleigh number is 657.5.

2. **A convective mixed layer**

   Suppose that nocturnal convection in the upper ocean is driven by a density difference $\Delta\rho/\rho_0 = 10^{-6}$ over the upper 40m (as in figure A.1). Compute the Rayleigh number, using the following values:

   $$\begin{aligned} \nu &= 1.0\times10^{-6}m^2s^{-1} \\ \kappa &= 1.4\times10^{-7}m^2s^{-1} \\ g &= 9.81ms^{-2} \end{aligned}$$

   Plot $\sigma(\tilde{k})$ for these parameter values. What is the horizontal wavelength $(2\pi/\tilde{k})$ of the fastest-growing instability? What are its growth rate and e-folding time? Give the e-folding time in hours, and compare it with the length of time over which convective conditions persist (say 12 hours). By what factor would the amplitude of this instability grow during that time?

3. **An unstable layer in an inviscid fluid**

   In a fluid with $\nu = \kappa = 0$, suppose that the mean buoyancy gradient has the following profile:

   $$B_z = B_{z0}(1 - 2\text{sech}^2 \alpha z). \quad \text{(A.0.3)}$$

   Sketch this function and show that the fluid is stably stratified except for an unstable layer surrounding $z = 0$.

   Solve (2.3.10) with boundary conditions $\hat{w} \to 0$ as $z \to \pm\infty$ for the special case $\tilde{k} = \alpha$. (Hint: try $\hat{w} = \text{sech}^2 \beta z$, where β is a constant to be determined.)

   In a later problem you will solve this numerically for a full range of $\tilde{k}$.