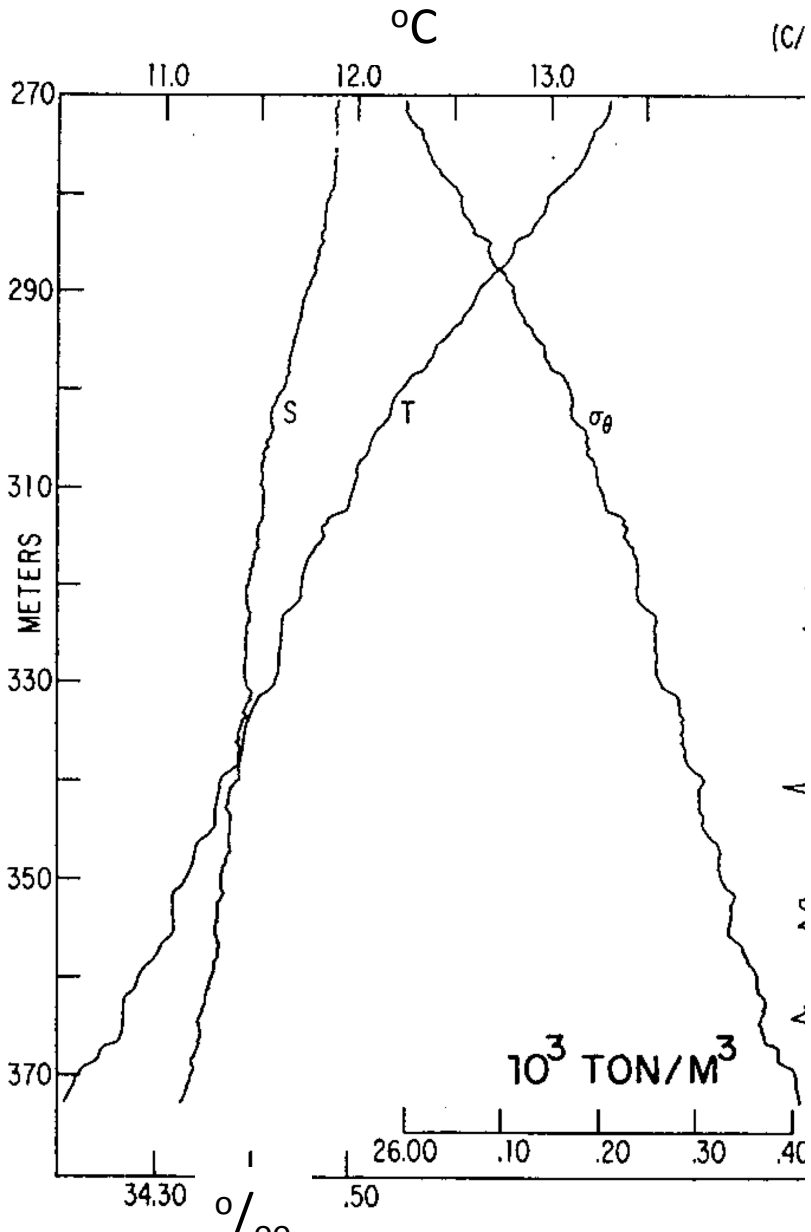
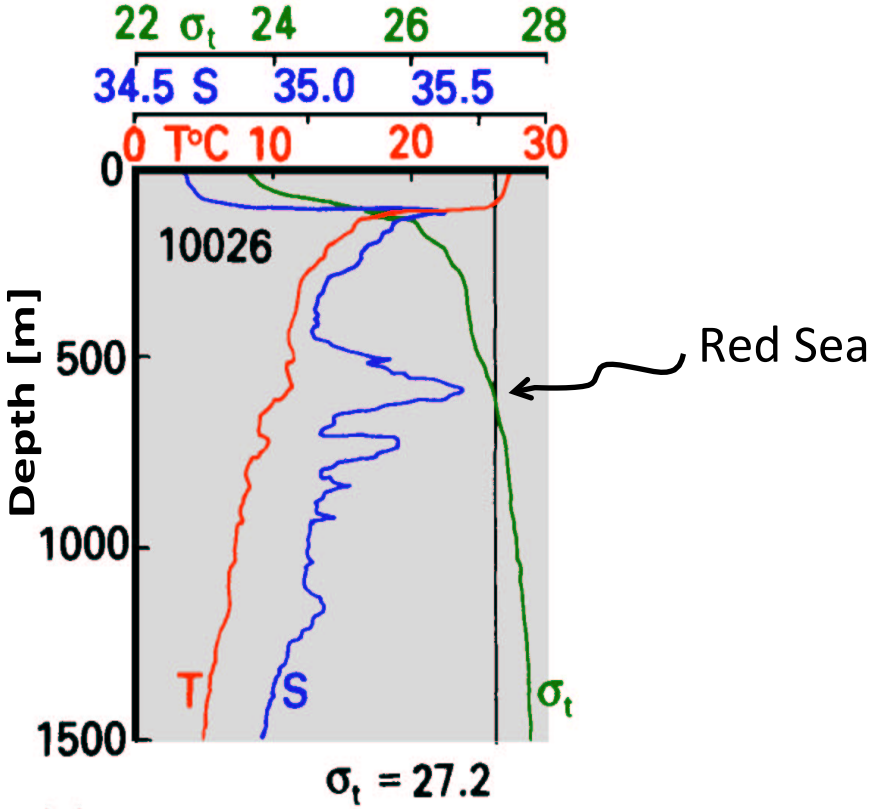
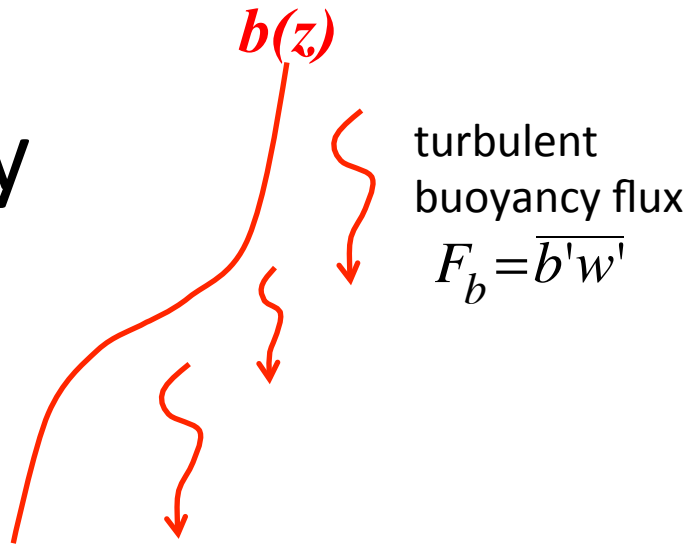
Layering instabities





The Phillips layering instability



$$\frac{\partial b}{\partial t} = -\frac{\partial F_b}{\partial z}$$

$$F_b = -K\frac{\partial b}{\partial z}$$

differentiate: $\frac{\partial b_z}{\partial t} = \frac{\partial^2}{\partial z^2}(Kb_z)$

$$\frac{\partial b_z}{\partial t} = \frac{\partial^2}{\partial z^2}(K b_z)$$

Linearize:

$$b_z = b_{z0} + b_z'(z,t)$$

$$K = K_0 + K^* b_z' \; ; \; K^* = \left(\frac{\partial K}{\partial b_z}\right)_{b_z = b_{z0}}$$

$$F = -K b_z = -(K_0 + K^* b_z')(b_{z0} + b_z') = -K_0 b_{z0} - K_0 b_z' - b_{z0} K^* b_z'$$

Substitute:

$$\frac{\partial b_z'}{\partial t} = \frac{\partial^2}{\partial z^2}\left[K_0 b_{z0} + (K_0 + b_{z0} K^*) b_z'\right] = (K_0 + b_{z0} K^*)\frac{\partial^2 b_z'}{\partial z^2}$$

$$\Rightarrow \frac{\partial b_z'}{\partial t} = \underbrace{(K_0 + b_{z0} K^*)}\frac{\partial^2 b_z'}{\partial z^2}$$

effective diffusivity

$$\frac{\partial b_z'}{\partial t} = (K_0 + b_{z0}K^*)\frac{\partial^2 b_z'}{\partial z^2}$$

Normal mode:

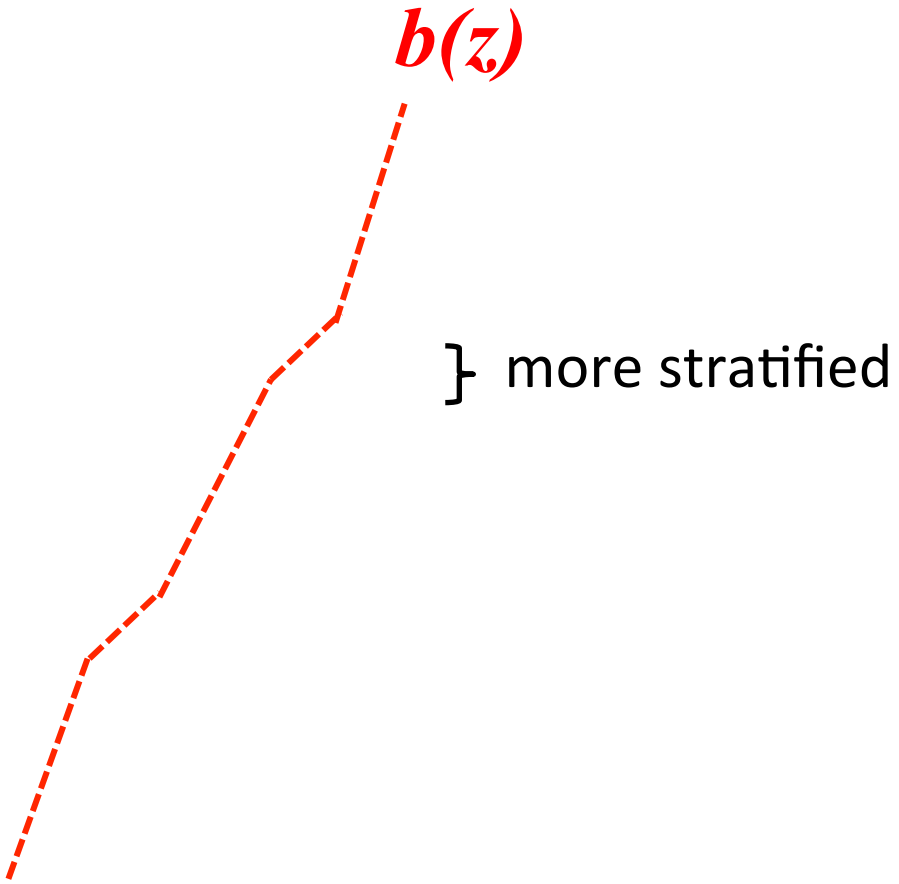
$$b_z = N_0 e^{\sigma t + \imath m z}$$

$$\Rightarrow \sigma N_0 = -m^2(K_0 + b_{z0}K^*)N_0$$

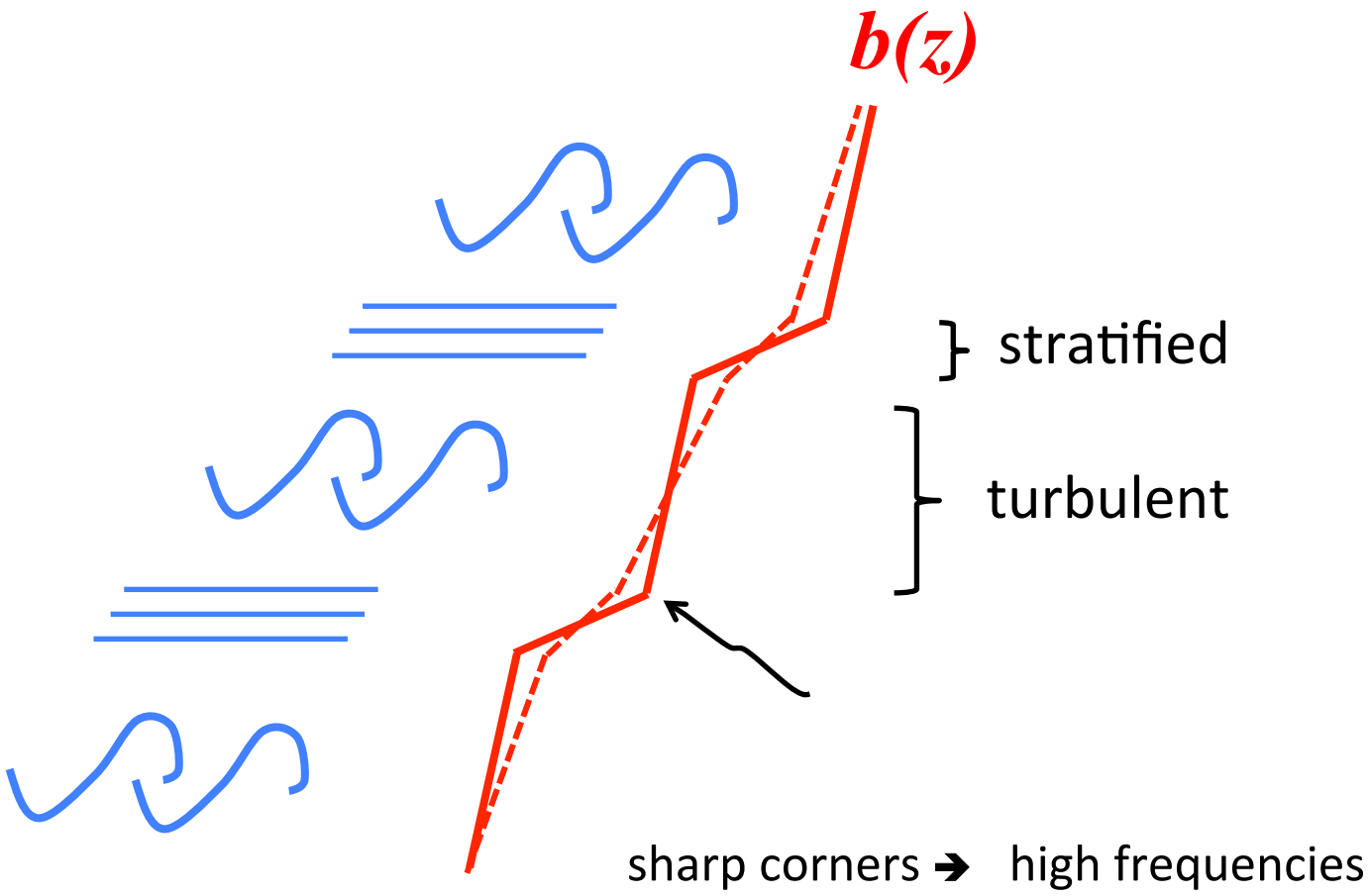
$$\Rightarrow \sigma = -m^2(K_0 + b_{z0}K^*)$$

If $K^* < -\frac{K_0}{b_{z0}}$ then $\sigma > 0$

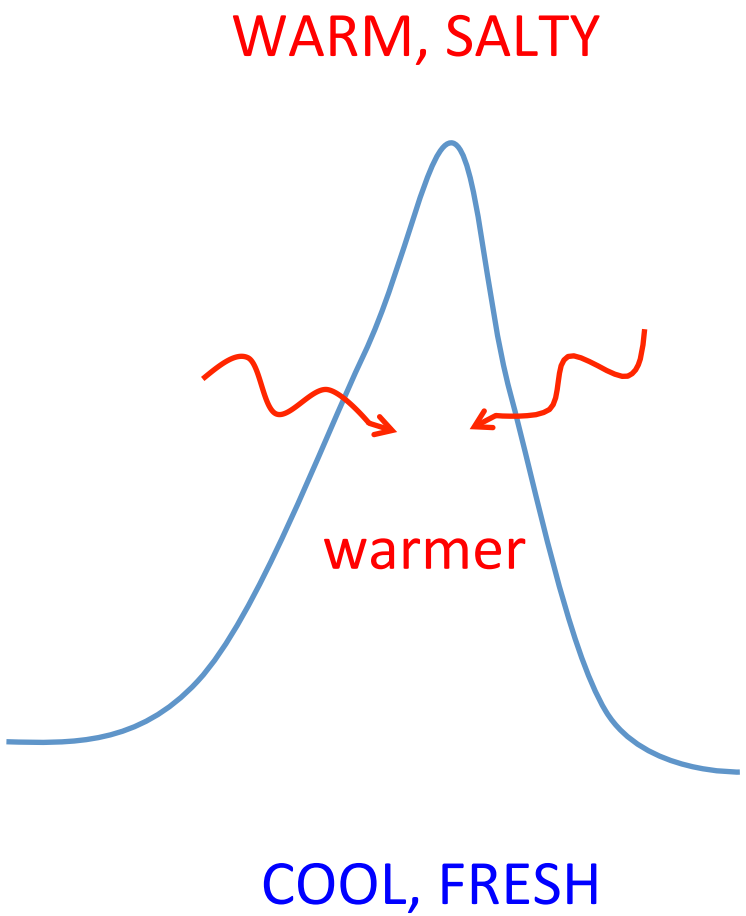
The Phillips layering instability



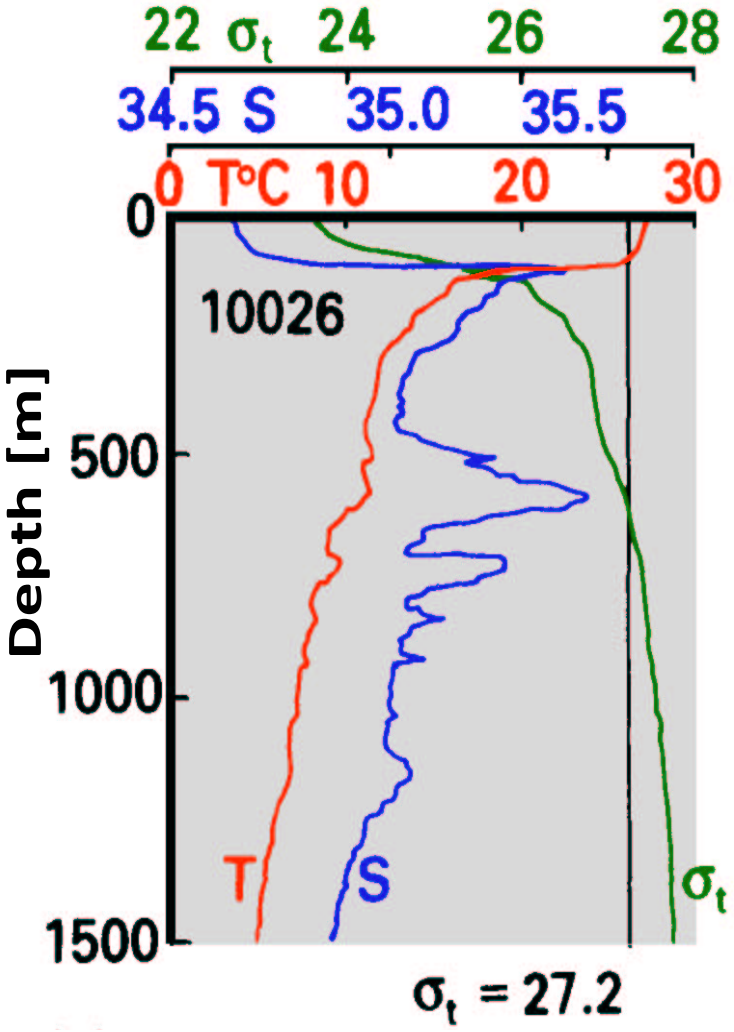
The Phillips layering instability



salt finger instability

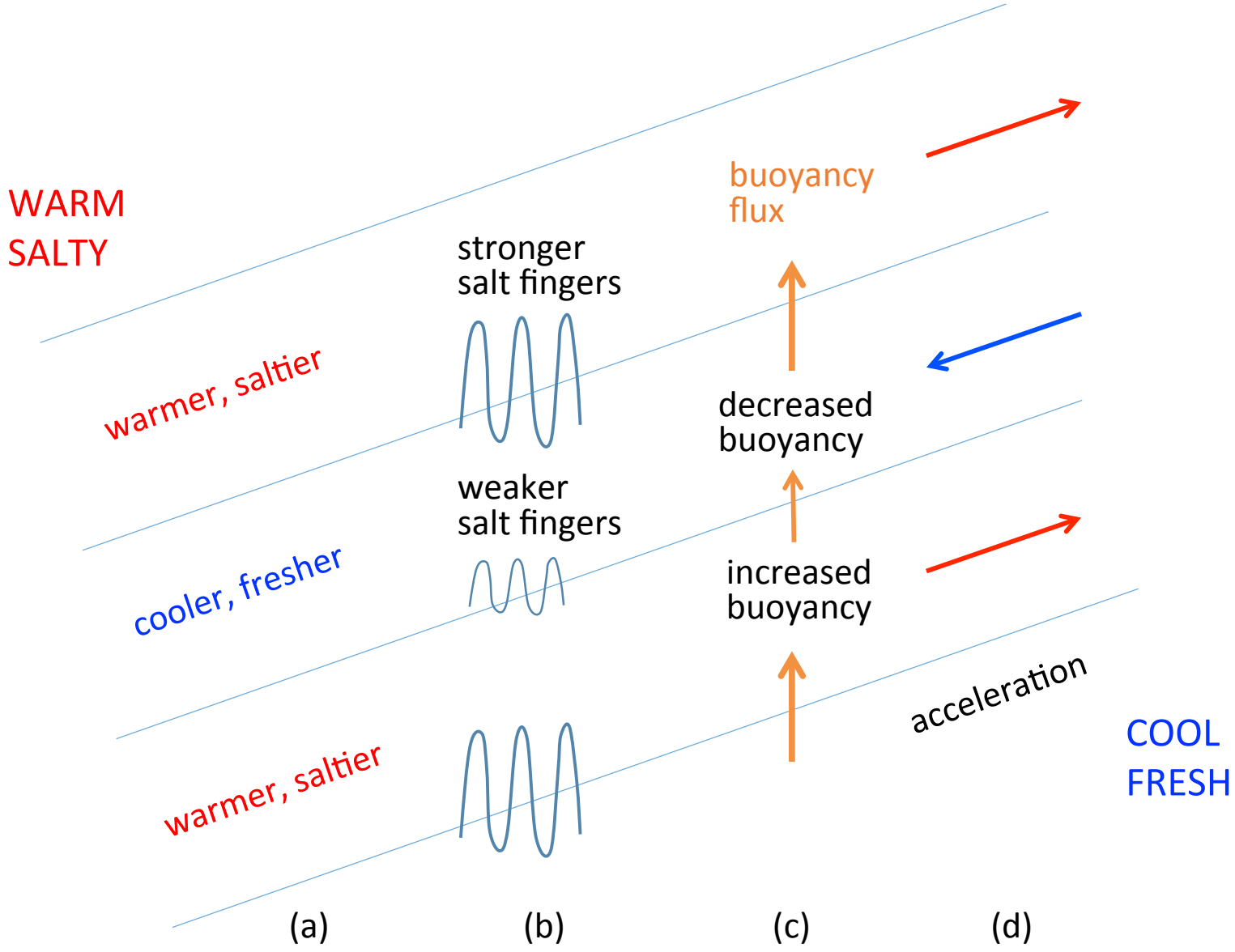


Interleaving in the Somali current



WARM, SALTY

COOL, FRESH


WARM
SALTY
warmer, saltier
cooler, fresher
warmer, saltier
stronger
salt fingers
weaker
salt fingers
buoyancy
flux
decreased
buoyancy
increased
buoyancy
acceleration
COOL
FRESH
(a)
(b)
(c)
(d)