# OC 670: Fluid Dynamics

Fall term, 2018, 4 credits

*No single thing abides, but all things flow.*
*- Heraclitus*

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**Class times**: M 1:00-2:20; W 1:00-2:20; F 1:00-1:50.

**Location**: Burt 176

**Text**: *All Things Flow: Fluid Mechanics for the Natural Sciences,* free online textbook. (Available in hardcopy.)

## Prerequisites:

There are no formal prerequisites, but the lectures assume familiarity with linear algebra up to eigenvectors and calculus up to div, grad and curl. A primer summarizing these topics will be provided.

### Course Objectives:

- Understanding flow: physical intuition and mathematical analysis
- Detailed exploration of some basic classes of flow

As a gauge of understanding, you should be able to estimate (for example):

- the depth of a flooding river based on its speed,
- the interval between sneaker waves at the beach,
- the pressure drop in a tornado given its wind speed,

and explain the assumptions that underlie your estimates.

## Activities:

In addition to lectures, students will spend at least (typically) 4 hours/week on assigned exercises and 4 hours/week on individual study (e.g. assigned reading, compiling lecture notes). Each student will lead at least one class lecture based on the text. The midterm exam is “take home”; the final is 2 hours, in class, lecture notes are permitted.

## Evaluation:

Weekly homework, W-W (50%).

Student-led lecture (10%).

Exams (40%)

- Midterm (take-home): Oct 22^th^-29^th^
- Final (open notes): Tues Dec. 4^th^, 12:00-2:00.

# Course Outline

## Part 1: Theory

**a) *Introduction***

- Some basic flow structures
- Normal and transverse stresses

**b) *Topics from linear algebra***

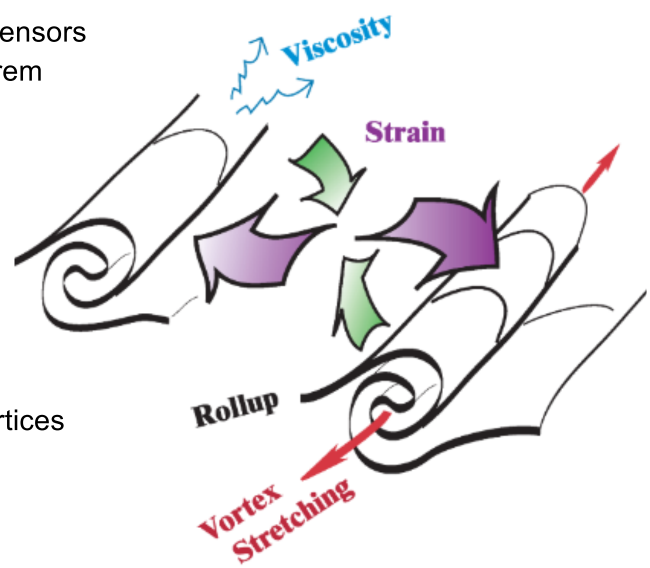
- Index notation: the Einstein convention, free/dummy indices
- Dot products, projections
- Matrix-vector multiplication, geometric interpretation
- Eigenvalues and eigenvectors

**c) *Cartesian tensors***

- Human perceptions of physical reality
- Cartesian position vectors, coordinate rotations
- Definition of a vector
- Torque, moment of inertia, and angular acceleration
- Scalars, vectors and higher-order tensors
- Tensor properties: symmetry, isotropy
- The alternating tensor: properties and applications
    - The cross product
    - The triple product
    - The determinant

**d) *Tensor calculus***

- Differential operations (div, grad, curl, etc.) applied to tensors
- Flux and divergence; the generalized divergence theorem
- Vorticity and circulation.




**e) *Fluid kinematics***

- Eulerian/Lagrangian descriptions of flow
- The streamfunction
- Relative motion near a point
    - The rotation tensor
        * Vortex motions in two dimensions
        * Vortex models: rigid, irrotational and  Rankine vortices
    - The strain tensor
        * Normal and transverse strains
        * Diagonalizing a 2$^{nd}$ order tensor
        * Principal axes, principal strains
- The "atom" of turbulence

**f) *Fluid dynamics***

- Contact forces and the stress tensor
    - Force on a coordinate plane
    - Is the stress "tensor" really a tensor? (force on a tetrahedron)
    - Net force on a fluid parcel

        - Symmetry of the stress tensor (torque on a cube)
    - Conservation of mass and momentum
        - Liebnitz' Rule
        - Conservation of mass
        - Conservation of momentum:
        - Hydrostatic equilibrium, pressure
    - Newtonian fluids
        - Stress-strain relation for Newtonian fluids
        - Navier-Stokes equations
    - Boundary conditions
    - Energy transfers in a Newtonian fluid
        - Kinetic energy
        - Potential energy
        - Internal energy, 1st law of thermodynamics
        - Temperature equations for water and air
    - Equations of state (ideal gas, seawater)
    - Useful approximations: incompressible, inviscid, Boussinesq

## Part 2: Application to selected flow geometries

**a) *Vortices***

- The homogeneous, inviscid case
    - Vortex filaments, Helmholtz' 1st theorem
    - Vortex stretching and tilting
    - Vortex tubes, Helmholtz' 2nd theorem
    - Kelvin's theorem
    - Vorticity in 2D flow
    - Examples: storm drains, hurricanes, dust devils and tornadoes, kolks in rivers and megafloods, vortex interactions in 2D turbulence, toroidal vortices
- Viscous effects
    - Burgers vortex
    - Vortices in turbulence, tornadoes, eddy viscosity
- Buoyancy effects
    - The baroclinic torque
    - The Boussinesq limit

**b) *Waves***

- Surface gravity waves
- Superpositions: sneaker waves, group velocity
- Short and long-wave approximations to the dispersion relation
- Hydrostatic flow

**c) *Hydraulic flows***

- The Froude number, flow over obstructions
- Hydraulic jumps in rivers and katabatic winds, the Missoula floods
- Breakers at the beach
- Turbulent river flow and flood waves